Reduced-Cost Construction of Jacobian Matrices for High-Resolution Inversions

Hannah Nesser[1], Daniel J. Jacob[1], Joannes D. Maasakkers[2], Tia R. Scarpelli[3], Melissa P. Sulprizio[1], Yuzhong Zhang[4], Chris H. Rycroft[1]

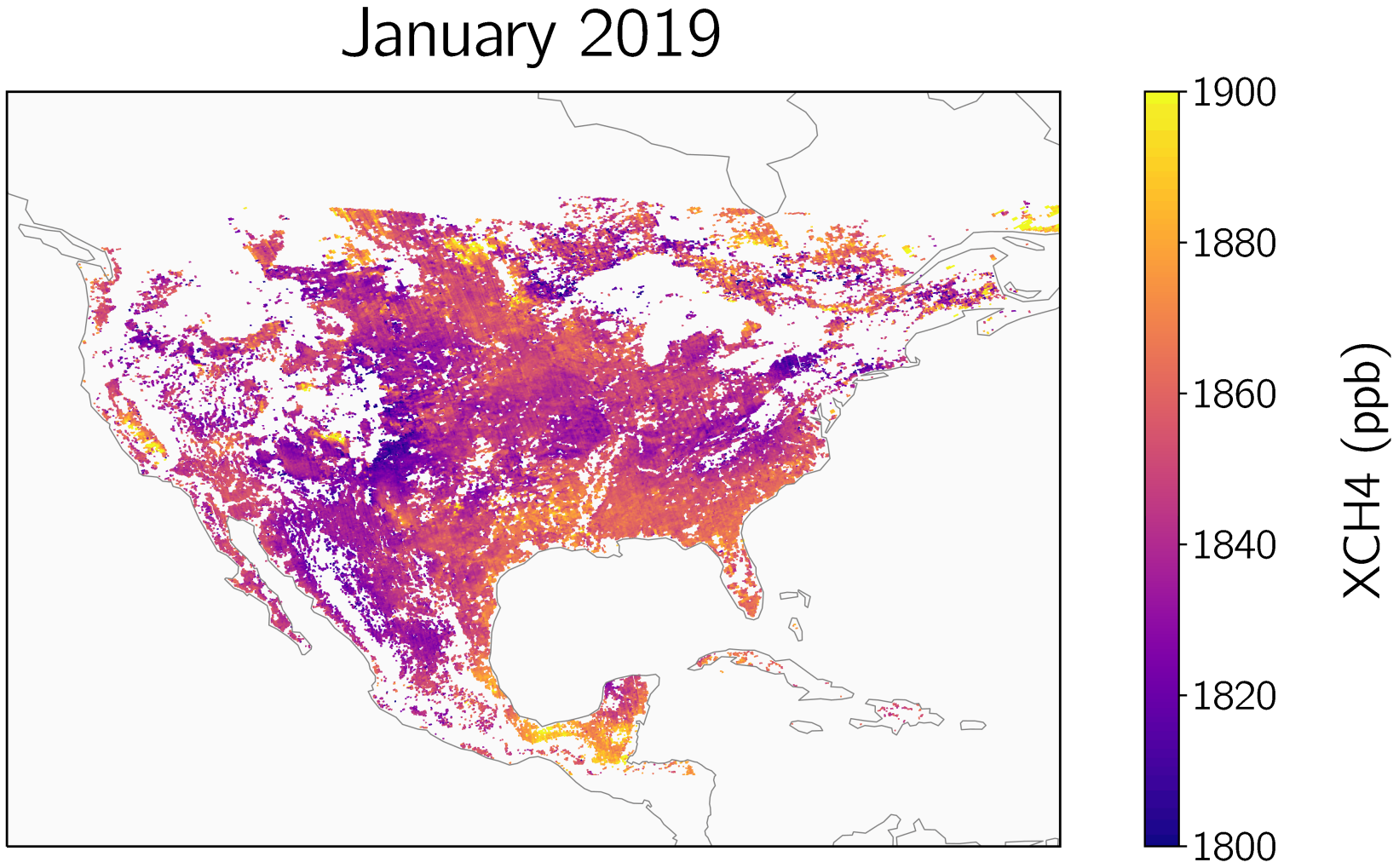
[1]School of Engineering and Applied Sciences, Harvard University, Cambridge, MA, USA

[2]SRON Netherlands Institute for Space Research, Utrecht, the Netherlands

[3]Department of Earth and Planetary Sciences, Harvard University, Cambridge, MA, USA

[4]School of Engineering, Westlake University, Hangzhou, Zhejiang Province, China

TROPOMI provides daily, global retrievals of atmospheric methane columns



An inversion uses atmospheric observations to improve constraints on emissions

emissions estimate (x_A, S_A)

modeled observations $(Kx_A + c)$

forward model

improved emissions estimate $(\hat{x}, \hat{S})$

- improved emission estimate
  $\hat{x} = x_A + \hat{S}K^T S_O^{-1}(y - (Kx_A + c))$
- improved error estimate
  $\hat{S} = (S_A^{-1} + K^T S_O^{-1} K)^{-1}$
- information content
  $A = I - \hat{S}S_A^{-1}$

minimize cost function

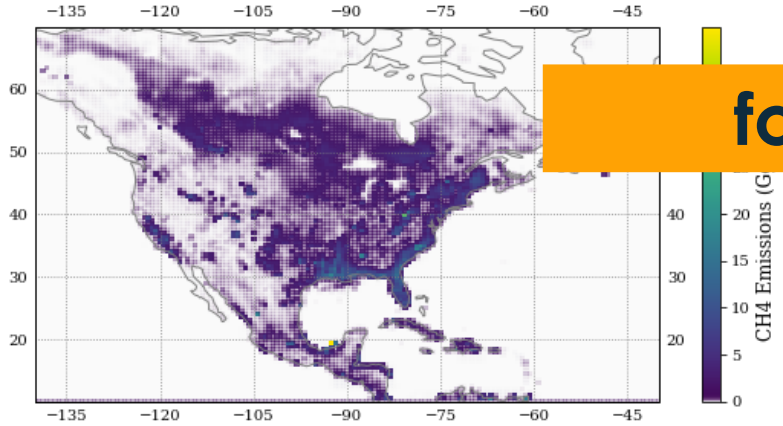
1. variational
2. analytical

observations (y, S_O)

minimization method:	variational	analytical
characterizes posterior error and information content		✓
finds true minimum of shallow cost function		✓
sensitivity tests require no significant additional computational cost		✓
computational cost is not limited by resolution	✓	

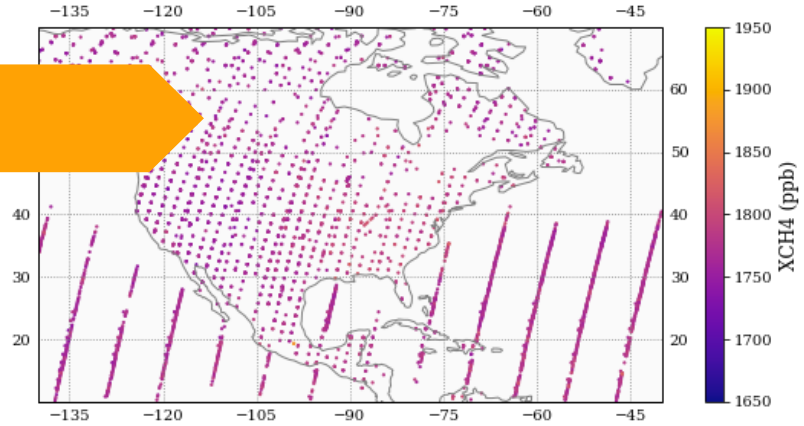
The computational cost of an analytical inversion is limited by resolution because of the construction of the Jacobian matrix

emissions estimate

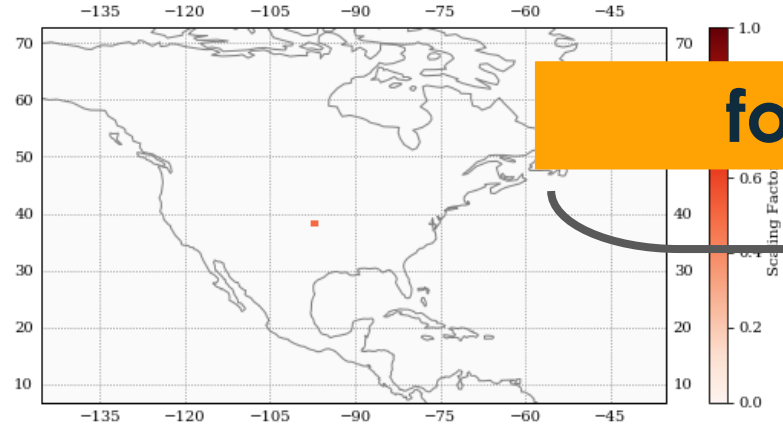


forward model

modeled observations

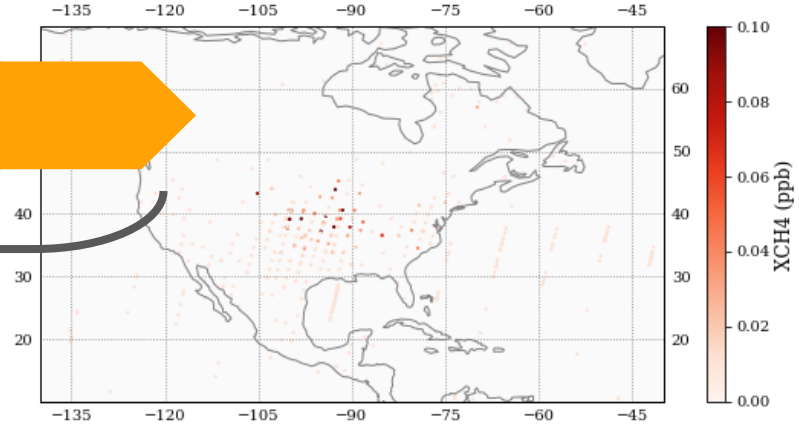


Δx



forward model

Δy

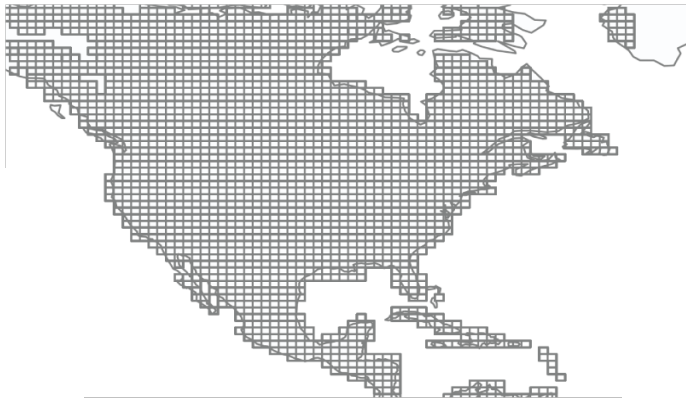


n perturbations,
n model runs

We can decrease the computational cost by optimally reducing the dimension or rank of the emission space

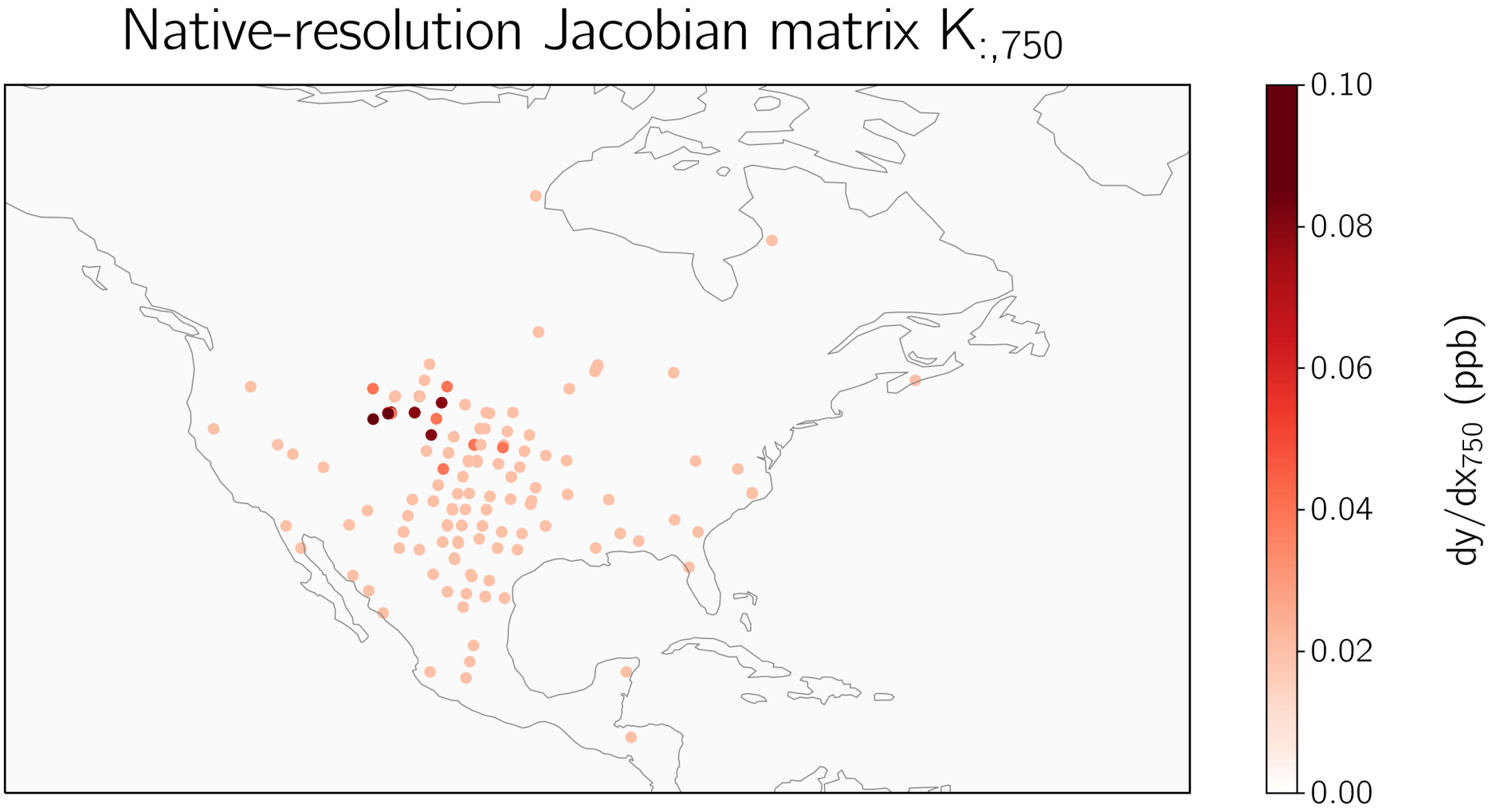
Native resolution

dimension n, rank $> k$



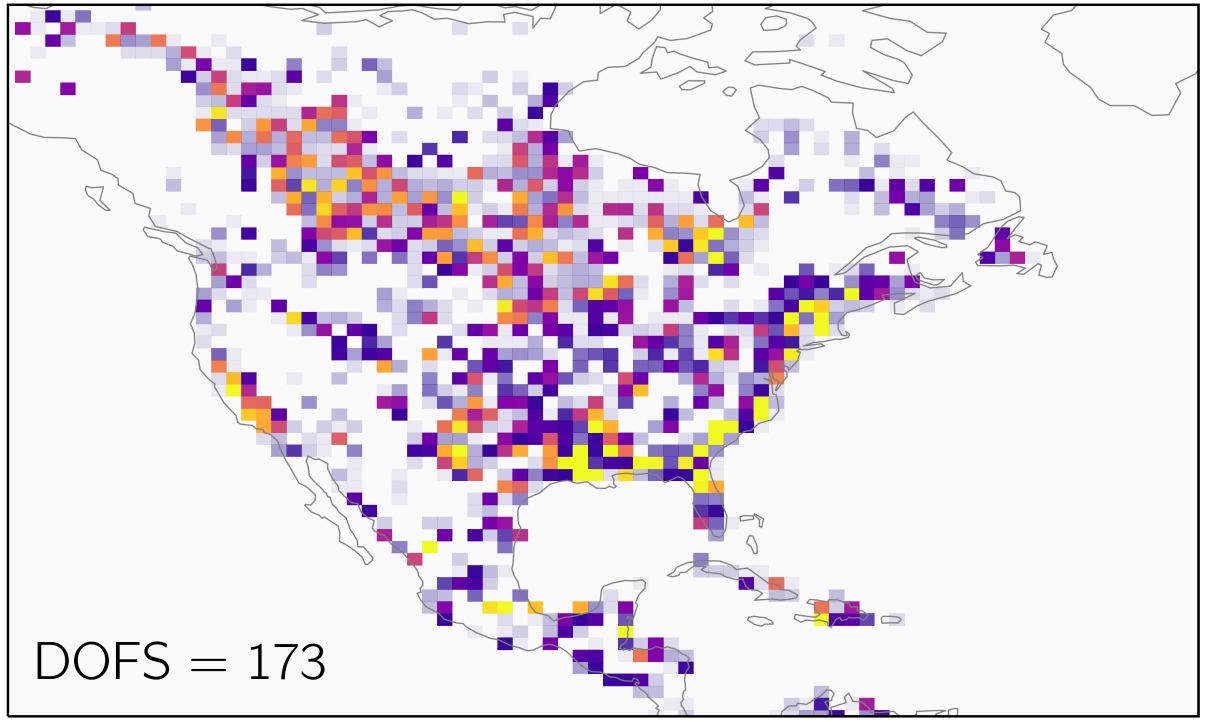
Nesser et al. (in review)

The Jacobian matrix is initialized by assuming that observations are most sensitive to local emissions



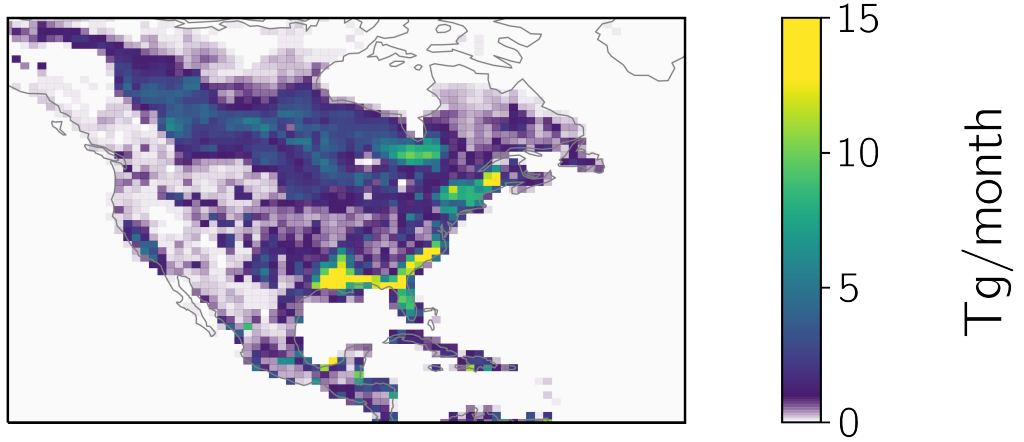
This estimate reproduces native-resolution information content because both depend on prior errors and observation density

Initial estimate averaging kernel sensitivities

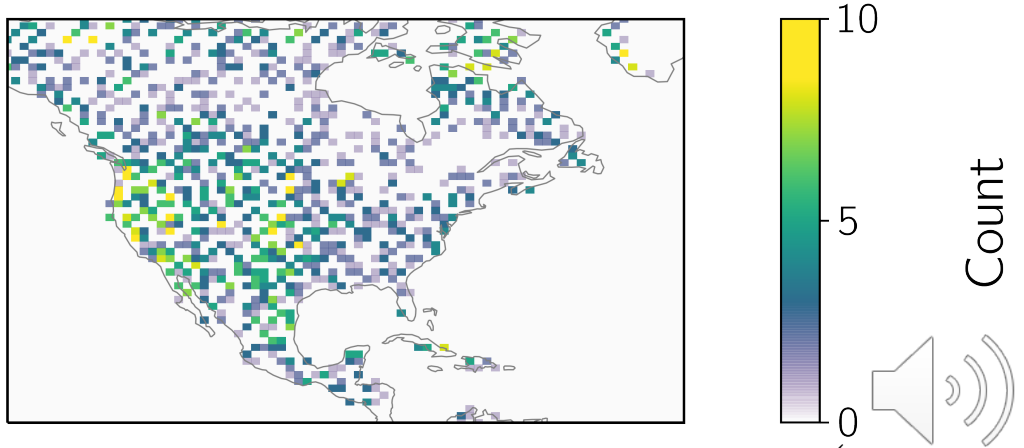


$\partial \hat{x}_i / \partial x_i$

Prior error standard deviation



GOSAT observation density (July 2009)



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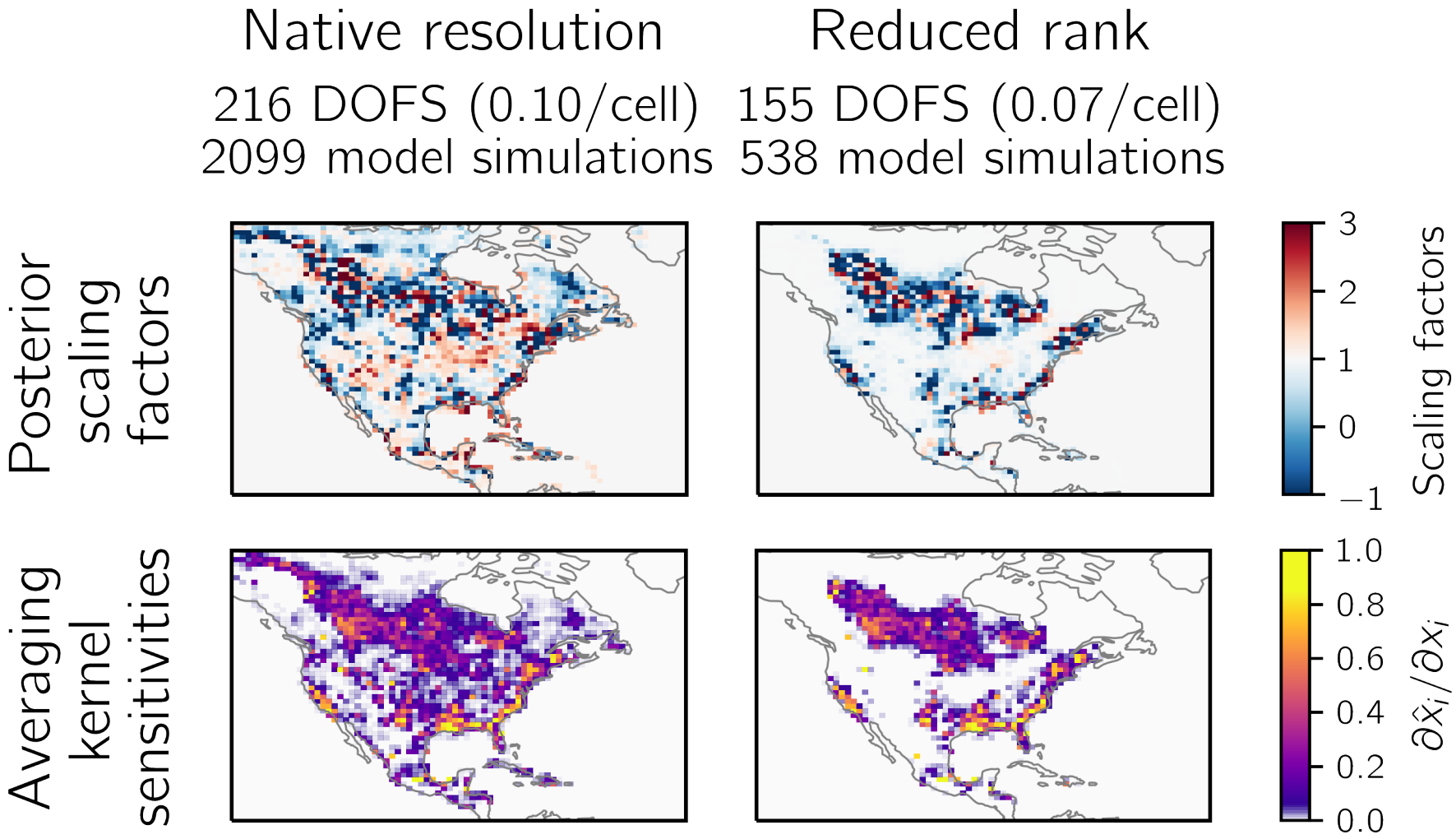
The resulting Jacobian matrix reproduces the inverse results obtained at native resolution at ¼ of the computational cost



Nesser et al. (in review)

We are using the reduced-rank method in an inversion of 2019 TROPOMI observations at 0.25° × 0.3125° over North America