

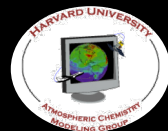


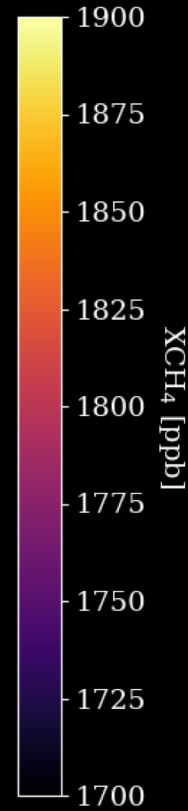
Reduced-Cost Construction of Jacobian Matrices for High-Resolution Inverse Modeling

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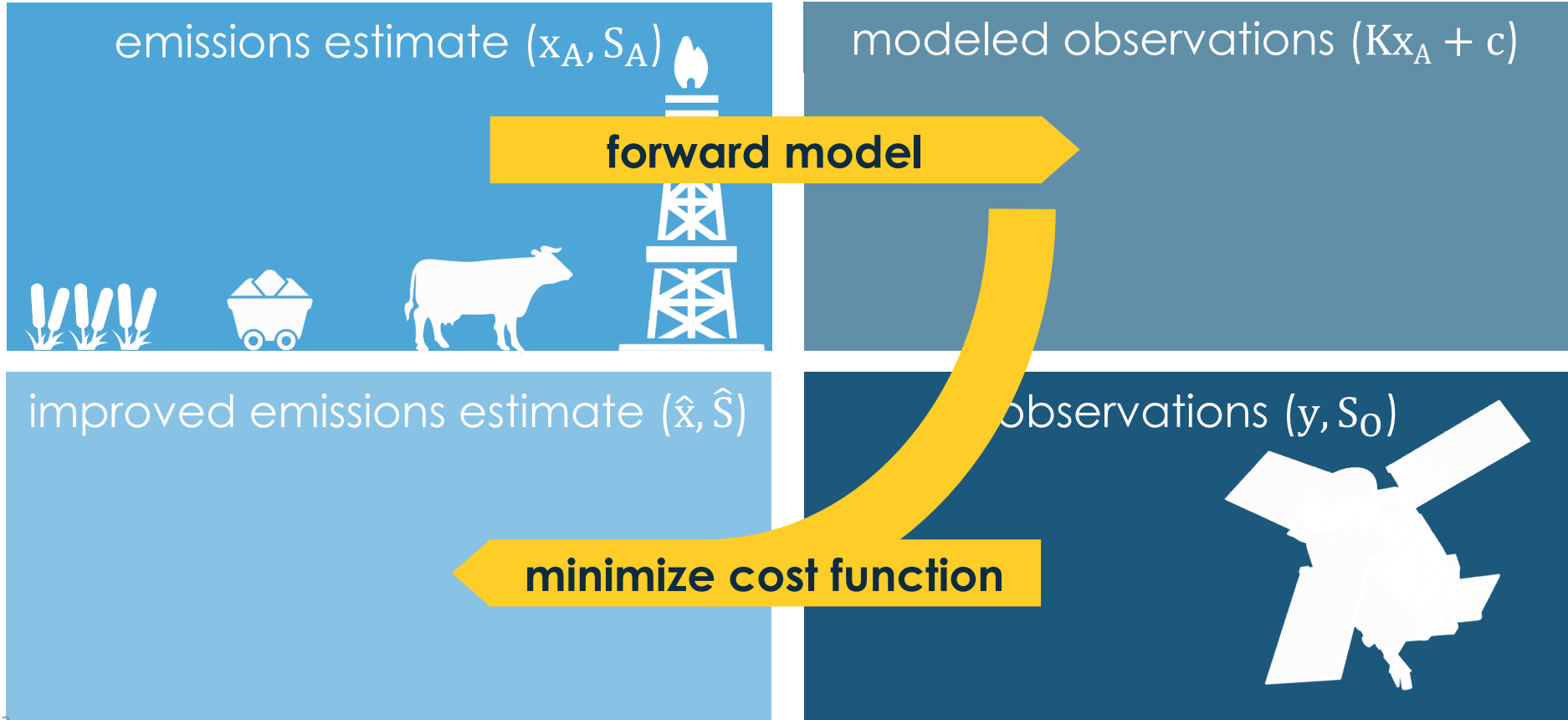
²SRON Netherlands Institute for Space Research, Utrecht, the
Netherlands





TROPOMI
provides daily,
global
retrievals of
atmospheric
methane
columns

Satellite retrievals can be used in inverse frameworks to improve constraints on emissions estimates



Minimization method:**Variational****Analytic**

Computational cost is not limited by resolution



Finds true minimum of shallow cost function



Characterizes posterior error and information content

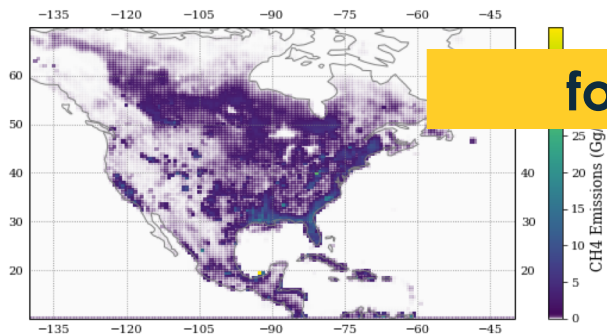


Sensitivity tests require no significant additional computational cost



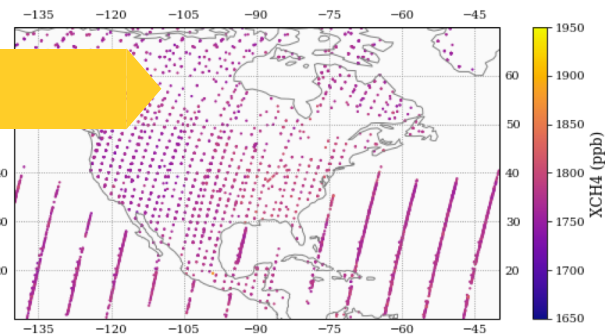
The computational cost of an analytic inversion is limited by constructing the Jacobian \mathbf{K}

emissions estimate

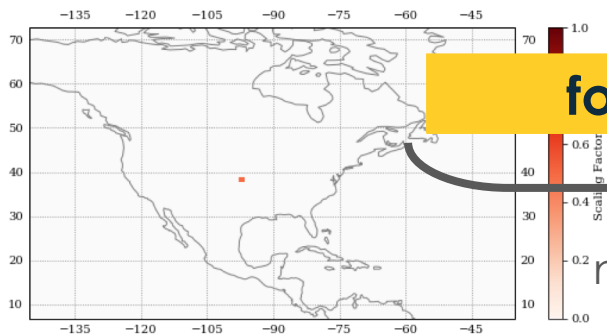


forward model

modeled observations



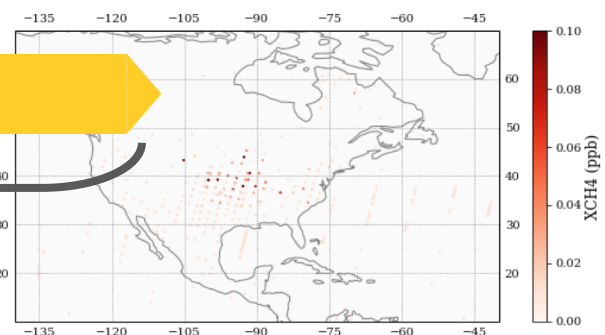
Δx



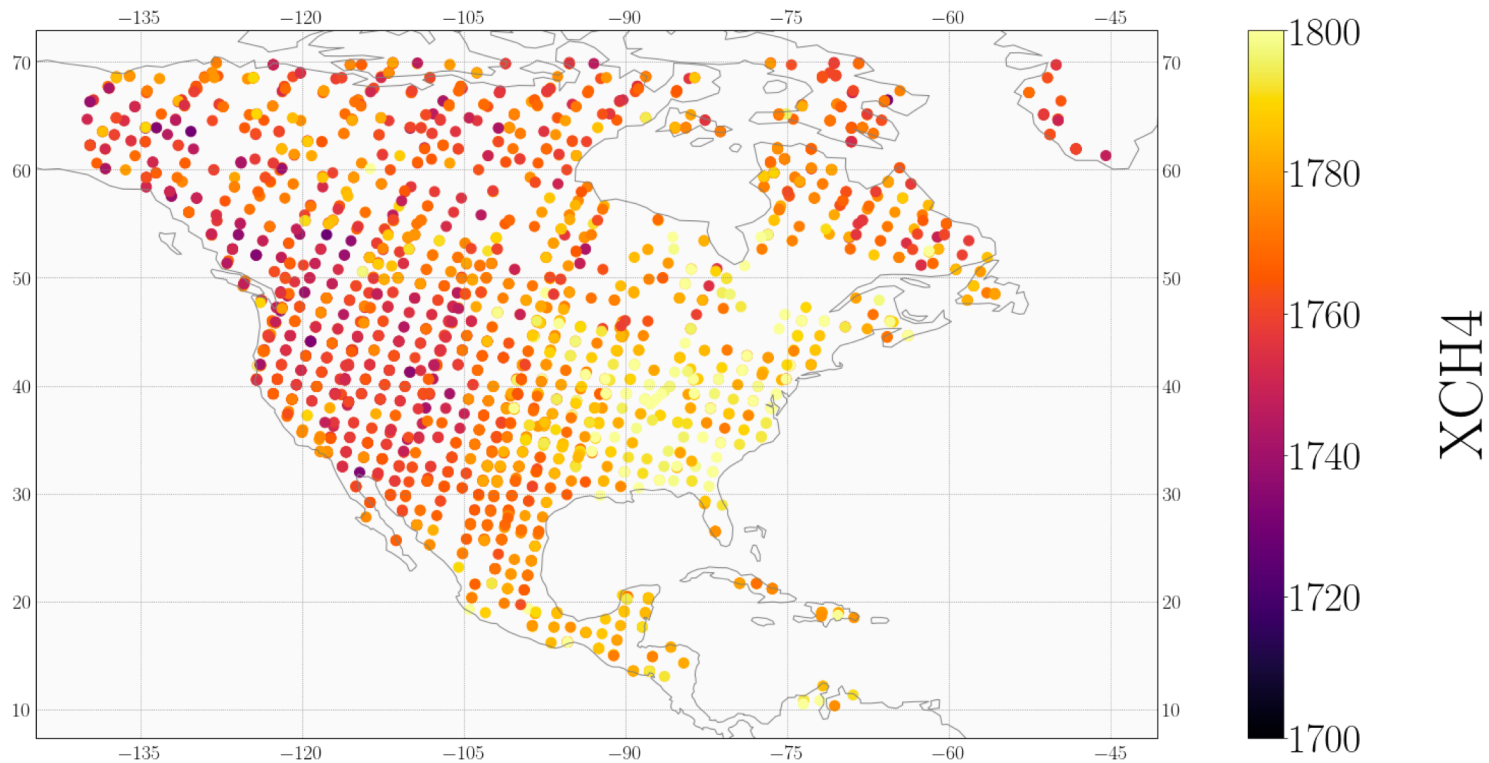
forward model

n perturbations,
 n model runs

Δy

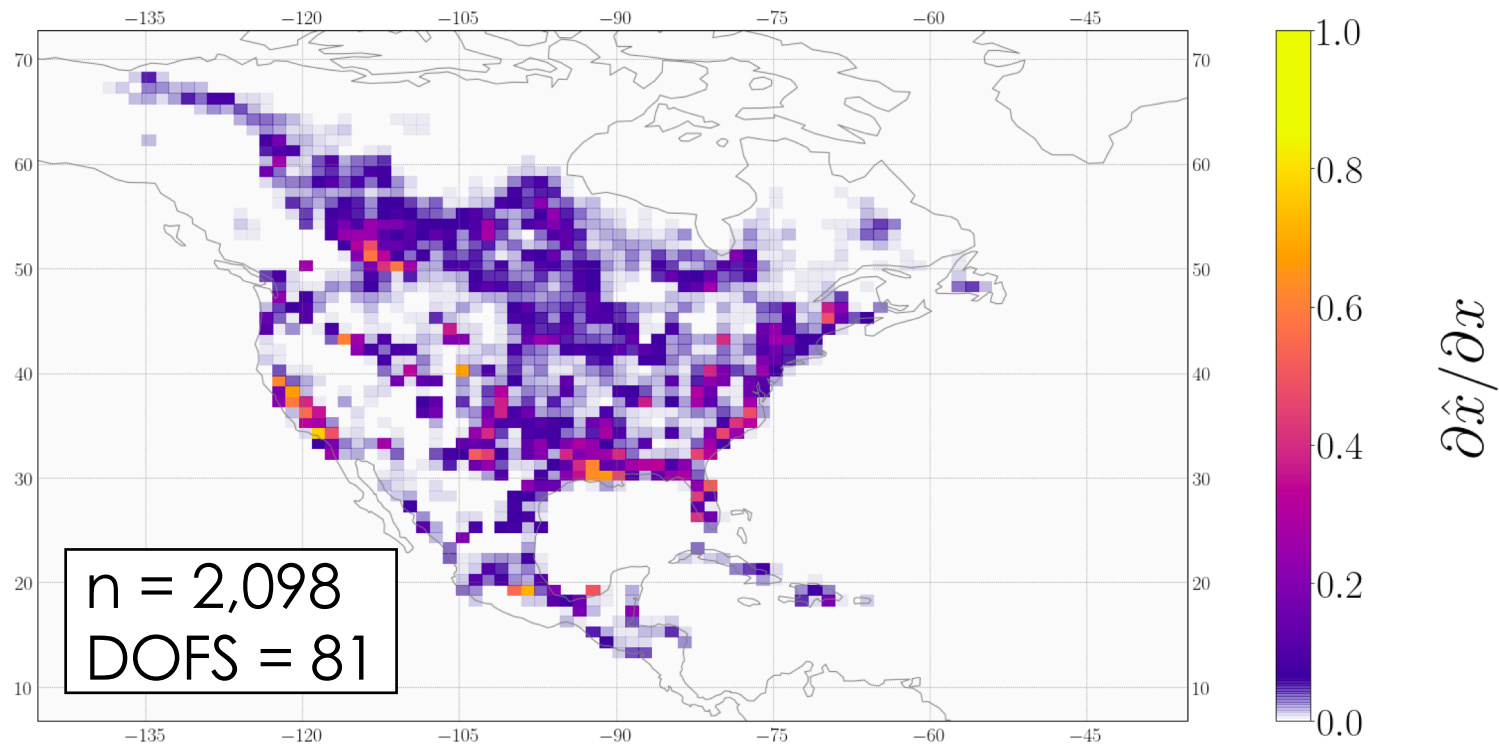


Analytic inversion of July 2009 GOSAT observations over North America at $1^\circ \times 1.25^\circ$ requires 2,099 model runs



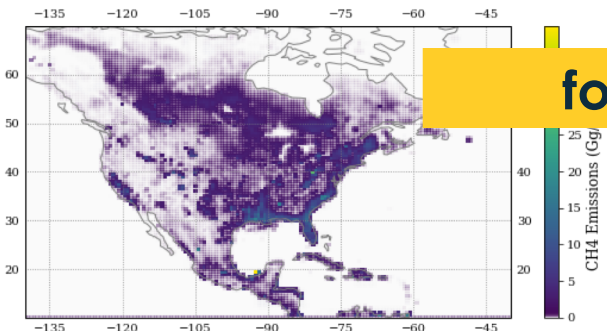
Analytic solution of the inversion shows that not all grid cells are equally well constrained

Averaging Kernel



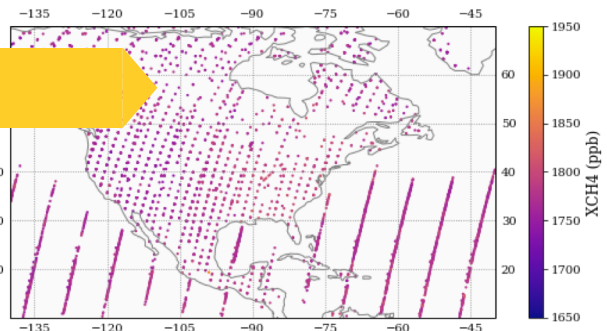
Perturbing the dominant patterns of information content would require $k < n$ model runs

emissions estimate

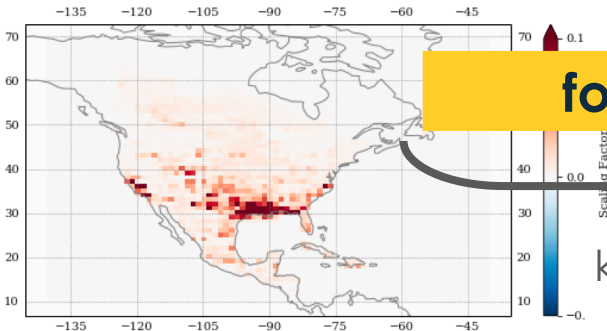


forward model →

modeled observations

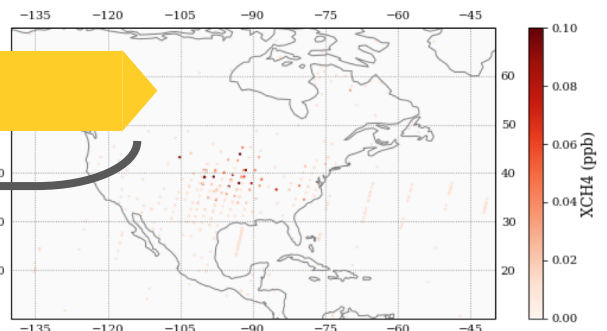


Δx



forward model →

Δy



k perturbations,
 k model runs

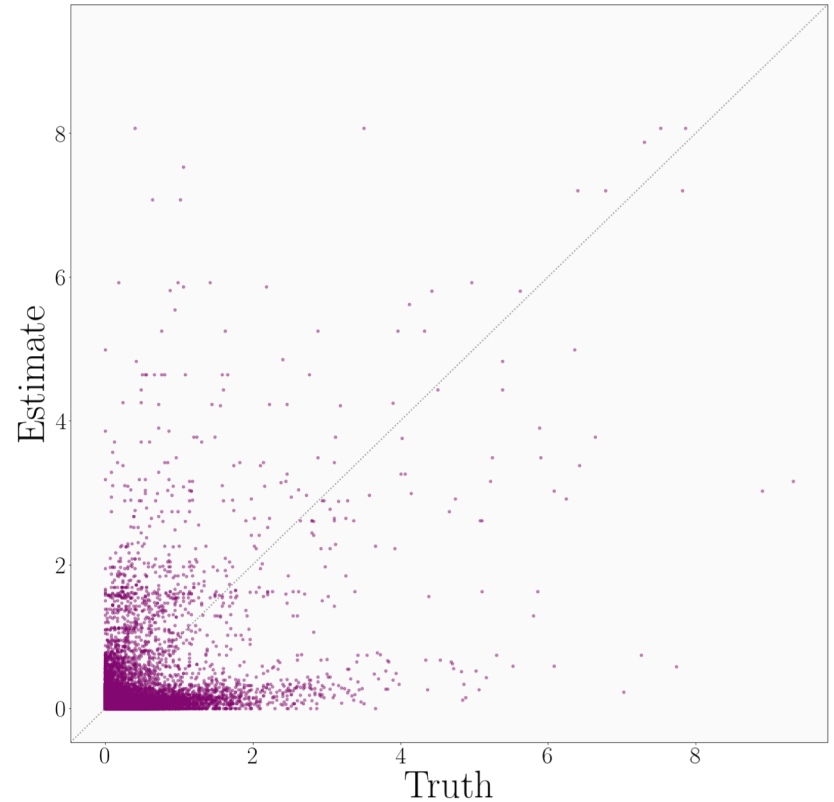
Initialize the Jacobian

Find the dominant patterns of information content

Perturb those patterns in the forward model

Transform the resulting Jacobian to the original resolution

Mass-Balance Jacobian

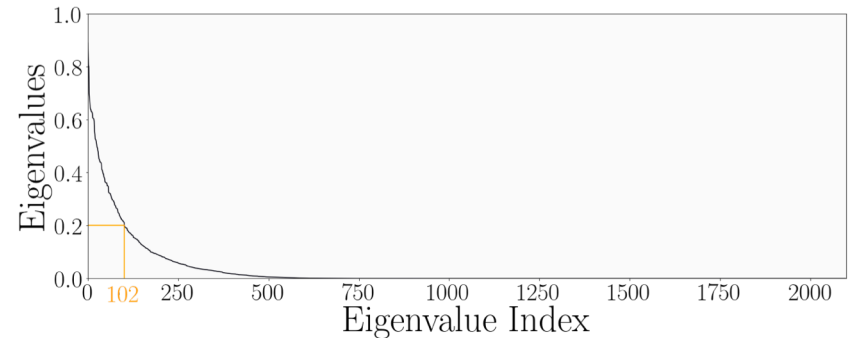
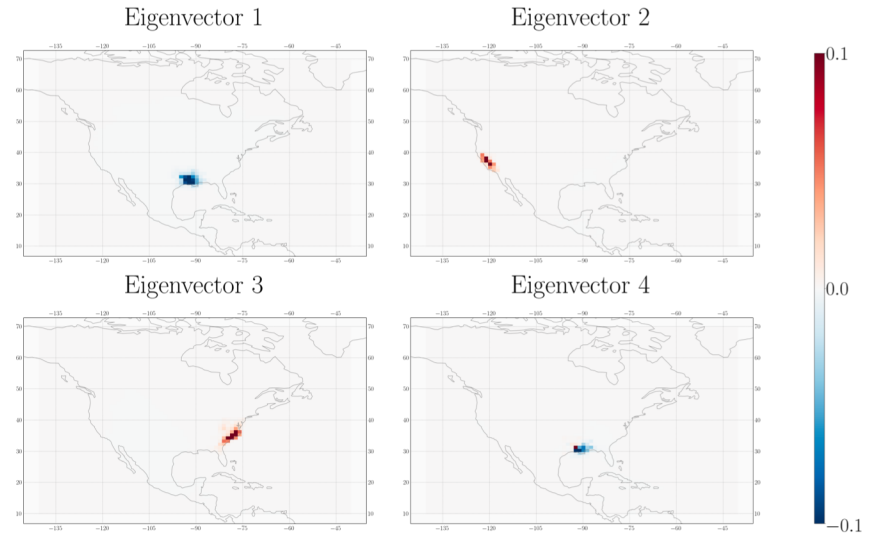


Initialize the Jacobian

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Transform the resulting Jacobian to the original resolution



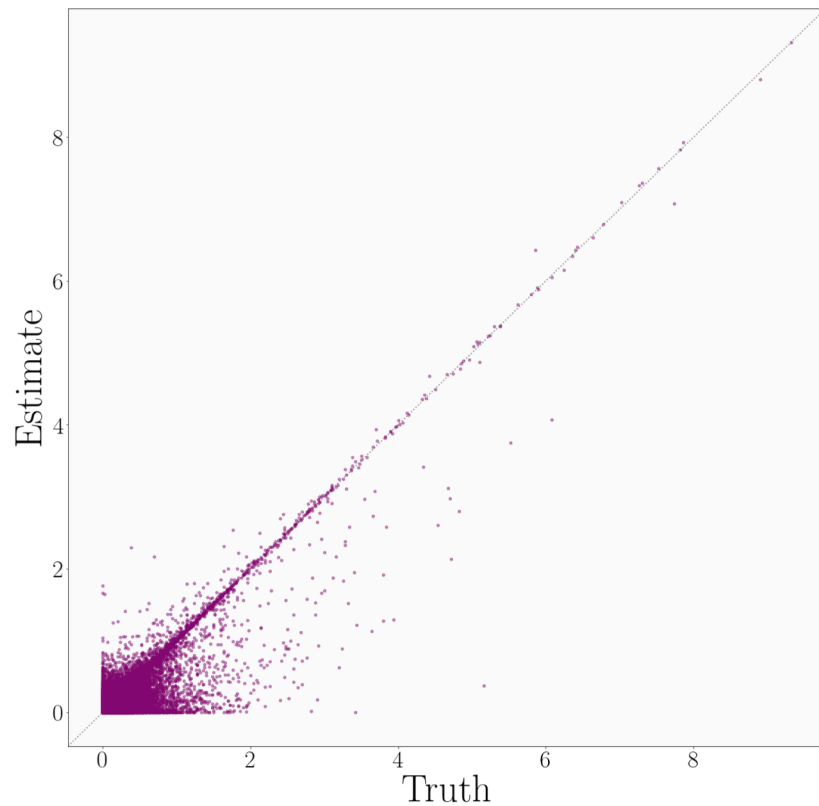
Initialize the Jacobian

Find the dominant
patterns of information
content

Perturb those patterns in
the forward model

Transform the resulting
Jacobian to the original
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Second Jacobian Update



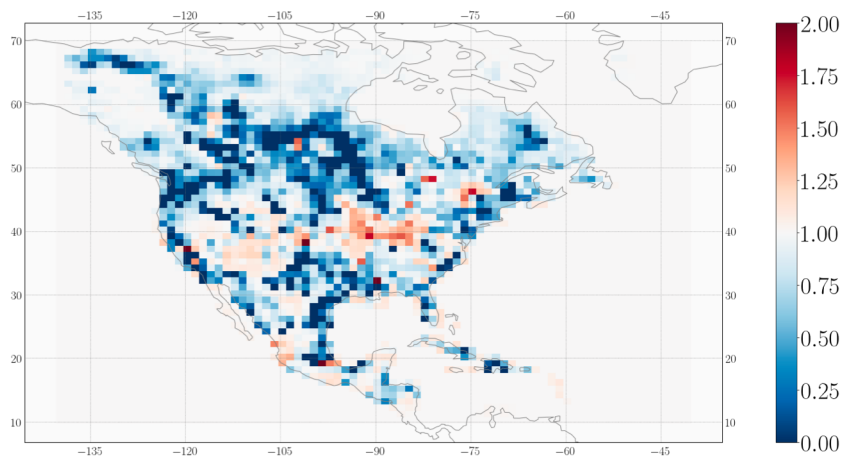
Can the low-rank Jacobian reproduce the true posterior solution?

2,099 model runs

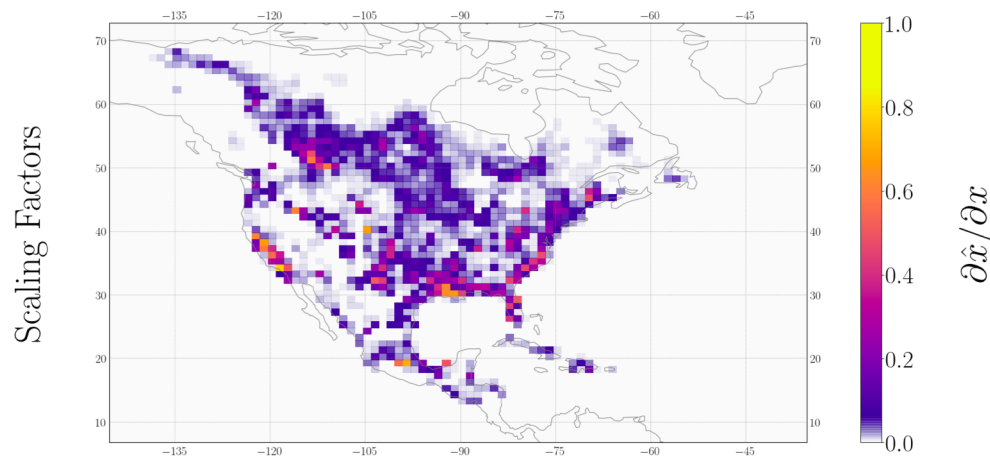
2,098 grid boxes

81 DOFS

True Posterior Mean



True Averaging Kernel



Scaling Factors

$\partial \hat{x} / \partial x$

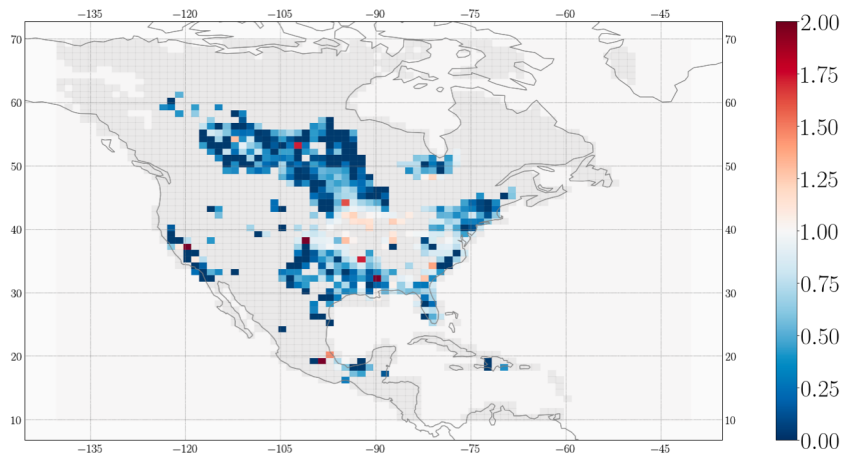
The low-rank Jacobian solves the inversion accurately in the grid cells with highest information content

306 model runs

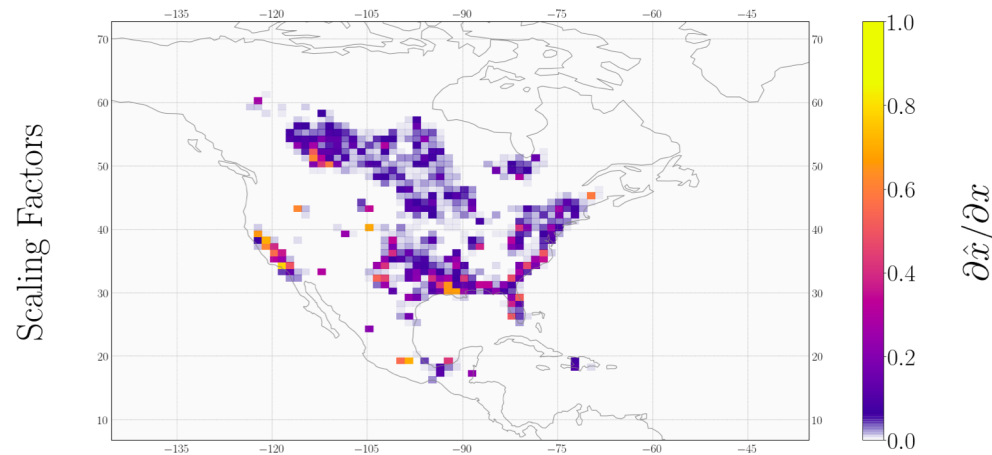
622 grid boxes

49 DOFS

Approximate Posterior Mean



Approximate Averaging Kernel

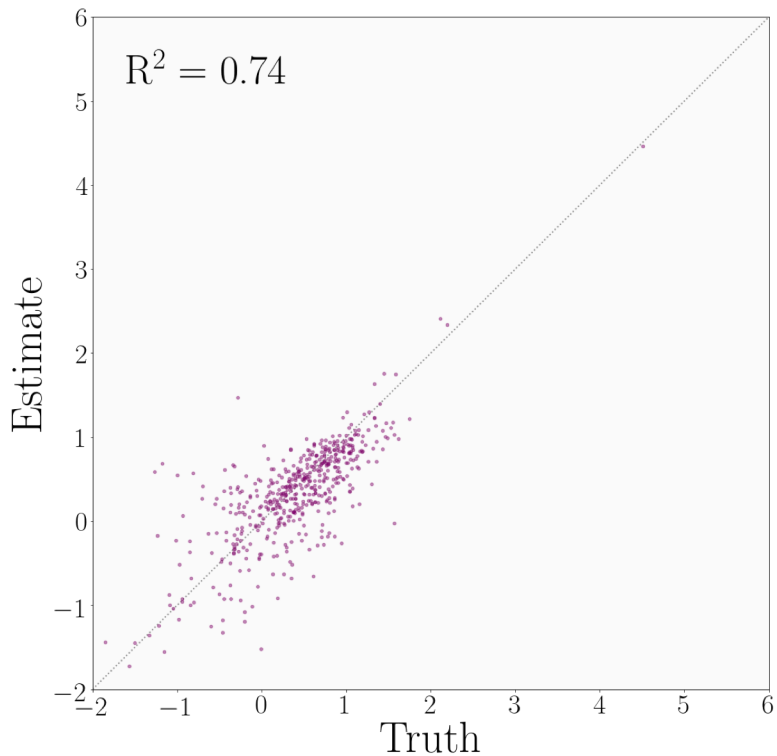


Scaling Factors

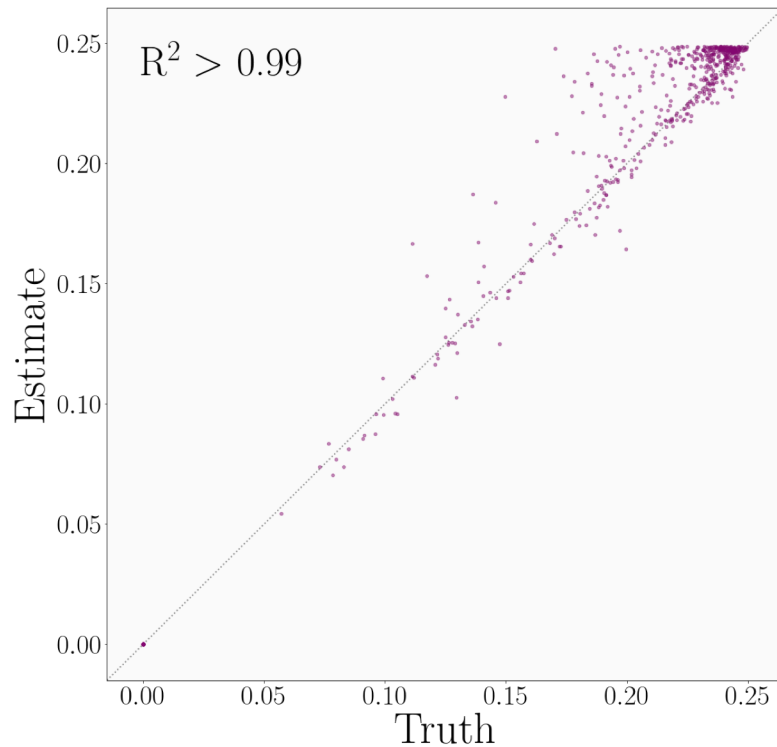
$\partial \hat{x} / \partial x$

The low-rank Jacobian solves the inversion accurately in the grid cells with highest information content

Posterior Mean



Posterior Variance



Low-rank Jacobian approximations significantly reduce the computational cost of analytic Bayesian inversions

- Perturbing the primary directions of information content can decrease the computational cost of constructing the Jacobian by an order of magnitude.
- The resulting Jacobian produces a posterior solution that is accurate in areas with sufficient information content.